# MATHEMATICS

# **Overview & Learning Goals Overview**

The mathematics major welcomes students from a broad range of backgrounds and driven by a diverse set of interests. It is designed with one common purpose: the pursuit of mathematics as a fundamental human endeavor with the ability to describe both the concrete and abstract aspects of the world around us. The power and utility of mathematics comes from its analytical approach to problem-solving. Our courses and requirements develop the ability to reason from hypothesis to conclusion, to analyze and solve quantitative problems, and to clearly and precisely articulate the underlying thought process. Students are encouraged to explore a broad range of courses offered by the mathematics department. It is central to our mission as a research and educational department to offer support and encouragement so that each member of our community can succeed and thrive.

# Learning Goals Understanding Higher Mathematics

- Understanding the need to transition from a procedural/ computational understanding of mathematics to a broader conceptual understanding encompassing logical reasoning, generalization, abstraction, and formal proof.
- 2. Understanding the **core of mathematical culture**: the value and validity of careful reasoning, of precise definition, and close argument. Understanding mathematics as a growing body of knowledge, driven by creativity, a search for fundamental structure and interrelationships, and a methodology that is both powerful and intellectually compelling.
- Understanding the fundamentals of mathematical reasoning and logical argument, including the role of hypotheses, conclusions, counterexamples, and other forms of mathematical evidence in the development and formulation of mathematical ideas.
- Understanding the methods and fundamental role of mathematics in modeling and solution of critical real-world challenges in science and social science.
- 5. Understanding the basic insights and methods of a broad variety of mathematical areas. All students of mathematics must achieve such understanding in calculus, naïve set and function theory, and linear algebra; and, ideally, will further achieve such understanding in probability and statistics, differential equations, analysis, and algebra.
- 6. Understanding in greater depth of at least one important subfield of mathematics such as abstract algebra, real analysis, geometry, topology, statistics, optimization, modeling, numerical methods, and dynamical systems.

# Skills Required for Effective Use of Mathematical Knowledge

- 1. **Problem-solving**—to develop confidence in one's ability to tackle difficult problems in both theoretical and applied mathematics; to translate between intuitive understandings and formal definitions and proofs; to formulate precise and relevant conjectures based on examples and counterexamples; to prove or disprove conjectures; to learn from failure; and to realize solutions are often multi-staged and require creativity, time, and patience.
- Modeling—to interactively construct, modify, and analyze mathematical models of systems encountered in the natural and social sciences; to assess a model's accuracy and usefulness; and to draw contextual conclusions from them.
- 3. **Technology**—to recognize and appreciate the important role of technology in mathematical work, and to achieve proficiency with the technological tools of most value in one's chosen area of concentration.
- 4. **Data and Observation**—to be cognizant of the uses of data and empirical observation in forming mathematical and statistical models, providing context for their use, and establishing their limits.
- Presentation—to produce clear, precise, motivated, and wellorganized expositions, in both written and oral form, using precise reasoning and genuine analysis.
- Mathematical Literature—to know how to effectively search the mathematical literature and how to appropriately combine and organize information from a variety of sources.

# Options for Majoring or Minoring in the Department

Students may elect to major in mathematics, the computer science and mathematics interdisciplinary major, the mathematics and economics interdisciplinary major, the mathematics and education interdisciplinary major, or to coordinate a major in mathematics with digital and computational studies, education, or environmental studies. Students pursuing coordinate or interdisciplinary majors may not normally elect a second major. Non-majors may elect to minor in mathematics.

Department Website (https://www.bowdoin.edu/math/)

# Faculty

Thomas Pietraho, *Department Chair* Suzanne M. Theberge, *Senior Department Coordinator* 

Professors: Adam B. Levy, Jennifer Taback‡, Mary Lou Zeeman Associate Professors: Christopher Chong, John D. O'Brien, Thomas Pietraho, Naomi Tanabe Assistant Professors: Raymond Maresca, Chandrika Sadanand Senior Lecturer: Eric C. Gaze

Visiting Faculty: Michael Ben-Zvi, Rhiannon Griffiths

Faculty/Staff Website (https://www.bowdoin.edu/math/faculty-and-staff/)

# Requirements Mathematics Major

A major consists of at least nine courses numbered 1800 or higher.

Code	Title	Credits
Required Courses		
Two core major courses		2
MATH 2000	Linear Algebra	
MATH 2020	Introduction to Mathematical Reasoning	
at least one advanced course (3000-3969)		1
Select at least six additional courses numbered 1751 or higher.		

Students who have mastered the material in MATH 2000 Linear Algebra prior to enrolling at Bowdoin may substitute another course numbered MATH 1751 or higher if they have received an appropriate placement. The same holds for MATH 2020 Introduction to Mathematical Reasoning if they also obtain approval of the department chair. All students must submit a planned program of courses to the department when they declare a major.

The requirement of an advanced course (3000–3969) is meant to ensure that all majors have sufficient experience in at least one specific area of mathematics as listed below:

Code	Title	Credits	
Algebra and Computation			
MATH 2301	Intermediate Linear Algebra		
MATH 2502	Number Theory and Cryptography		
MATH 2602	Group Theory		
MATH 2702	Rings and Fields		
MATH 3602	Advanced Topics in Group Theory		
MATH 3702	Advanced Topics in Rings and Number Theory $% \left( {{{\rm{A}}} \right)$		
Analysis			
MATH 2303	Functions of a Complex Variable		
MATH 2603	Introduction to Analysis		
MATH 3303	Advanced Complex Analysis		
MATH 3603	Advanced Analysis		
Geometry and Top	pology		
MATH 2404	Geometry		
MATH 3204	Тороlоду		
MATH 3404	Advanced Topics in Geometry		
Combinatorics and Probability			
MATH 2206	Probability		
MATH 2601	Combinatorics and Graph Theory		
Statistics and Ma	Statistics and Machine Learning		
MATH 1756	Data Science		
MATH 2606	Statistics		
MATH 2805	Mathematical principles of machine learning		
MATH 3606	Advanced Topics in Probability and Statistics		
Differential Equat	ions and Modeling		
MATH 1758	Biomathematics		
MATH 2208	Ordinary Differential Equations		
MATH 3108	Advanced Topics in Modeling		
MATH 3208	Advanced Topics in Dynamical Systems		

MATH 3209	Partial Differential Equations	
Operations Research		
MATH 2109	Optimization	
MATH 2209	Numerical Methods	
MATH 3109	Optimal Control	

# **Mathematics Minor**

A minor in mathematics consists of a minimum of five courses numbered 1751 or higher.

# **Interdisciplinary Majors**

The department participates in the interdisciplinary major programs of computer science and mathematics and mathematics and economics and mathematics and education. See the Interdisciplinary Majors (https://bowdoin-public.courseleaf.com/departments-programs/ interdisciplinary-majors/).

# Additional Information Additional Information and Department Policies

- Each of the courses required for the major or minor must be taken for a regular letter grade, not Credit/D/Fail, with a minimum earned grade of C-.
- At most, two of the nine courses required for the major, or one of the five courses required for the minor, can be transfer credits from other institutions.
- Independent studies and honors projects can count toward major and minor requirements with prior departmental approval. The department does not restrict the number of independent study and honors projects which count toward the major.
- Advanced Placement and International Baccalaureate scores, in addition to the mathematics placement questionnaire, are only used for placement.
- Courses from other departments or programs do not count toward the major or minor, but students may use mathematics courses toward another major or minor if that department or program allows.

# **Recommended Courses**

Regardless of mathematical interests, students are encouraged to take a wide variety of courses in theoretical and applied mathematics as well as statistics. We offer the following suggestions for students exploring different career pathways open to mathematics majors. We encourage all majors and minors to develop a strong relationship with their departmental advisor as they formulate a curriculum for their goals.

# **Secondary Education**

 Obtaining a broad base of knowledge in mathematics is important for secondary education, and recommended courses are listed as part of the m (https://bowdoin-public.courseleaf.com/departmentsprograms/interdisciplinary-majors/#requirementstext)athematics and education interdisciplinary major (https://bowdoinpublic.courseleaf.com/departments-programs/interdisciplinarymajors/#requirementstext). There are two formal ways to combine interests in mathematics and education at Bowdoin, namely the mathematics and education interdisciplinary major, and the coordinate major with education. However, neither is formally required to pursue a career in secondary education.

### **Actuarial Mathematics**

 Actuaries are leading professionals in finding ways to manage risk. In addition to courses in finance, economics, and computer science, students interested in actuarial science should include the following courses in their major. MATH 1800 Multivariate Calculus, MATH 2000 Linear Algebra, MATH 2206 Probability, MATH 2606 Statistics.

### Engineering

 Mathematics provides a firm foundation for students interested in a degree in engineering and for participation in one of the shared studies programs between Bowdoin College and other institutions. For more information about the recommended coursework and its timing, please see the Engineering Dual-Degree Options section under Special Areas of Study (https://bowdoinpublic.courseleaf.com/departments-programs/interdisciplinarymajors/#specialareasofstudytext).

#### **Finance, Economics, and Operations Research**

 Students interested in the fields of finance, economics, and operations research are encouraged to explore the courses required for the m (https://bowdoin-public.courseleaf.com/departmentsprograms/interdisciplinary-majors/#requirementstext)athematics and economics interdisciplinary major (https://bowdoinpublic.courseleaf.com/departments-programs/interdisciplinarymajors/#requirementstext).

#### **Graduate Study in Mathematics**

 Students interested in graduate study in mathematics should thoroughly explore the multiple perspectives present among courses offered by the department. Both breadth and depth in coursework are valuable. Ideally, MATH 2000 Linear Algebra and MATH 2020 Introduction to Mathematical Reasoning should be completed early in the major in order to meet the prerequisites of advanced courses. Students interested in graduate study in mathematics are encouraged to form close relationships with faculty in the department to receive mentoring and advice on their studies.

### **Statistics**

 Students interested in pursuing studies in statistics or biostatistics are encouraged to enroll in courses from the following list:
MATH 2000 Linear Algebra, MATH 2020 Introduction to Mathematical Reasoning, MATH 2206 Probability, MATH 2209 Numerical Methods, MATH 2301 Intermediate Linear Algebra, MATH 2603 Introduction to Analysis, MATH 2606 Statistics, MATH 2805 Mathematical principles of machine learning, MATH 3606 Advanced Topics in Probability and Statistics.

### **Information Security**

 Mathematics is at the core of modern information security research, including cryptography and network analysis. Students interested in this field are encouraged to obtain a solid foundation in both theoretical and applied mathematics, supplemented with courses in the computer science department. Courses to support an interest in this field include the following: MATH 2000 Linear Algebra, MATH 2020 Introduction to Mathematical Reasoning, MATH 2206 Probability, MATH 2301 Intermediate Linear Algebra, MATH 2502 Number Theory and Cryptography, MATH 2601 Combinatorics and Graph Theory.

### **Data Science**

 Understanding large data sets and drawing inferences and conclusions from their structure rely on an increasing variety of mathematical skills. The following courses in both theoretical and applied mathematics as well as statistics form a solid mathematical foundation in this area. MATH 1756 Data Science, MATH 2000 Linear Algebra, MATH 2020 Introduction to Mathematical Reasoning, MATH 2206 Probability, MATH 2301 Intermediate Linear Algebra, MATH 2601 Combinatorics and Graph Theory, MATH 2603 Introduction to Analysis, MATH 2606 Statistics, MATH 2805 Mathematical principles of machine learning, MATH 3606 Advanced Topics in Probability and Statistics. Students are also encouraged to complete coursework in computer science (https://bowdoinpublic.courseleaf.com/departments-programs/computer-science/).

### **Theoretical Computer Science**

 Students interested in a mathematical foundation complementing their studies in theoretical computer science are encouraged to explore courses from the following list: MATH 2000 Linear Algebra, MATH 2020 Introduction to Mathematical Reasoning, MATH 2206 Probability, MATH 2209 Numerical Methods, MATH 2301 Intermediate Linear Algebra, MATH 2502 Number Theory and Cryptography, MATH 2601 Combinatorics and Graph Theory, MATH 2602 Group Theory, MATH 3602 Advanced Topics in Group Theory.

# Information for Incoming Students (p. 3) Understanding Mathematics Placements

- Visit Bowdoin's Classfinder (https://classfinder.bowdoin.edu/ classfinder/) for a description of all courses currently offered.
- Students considering a major in economics, psychology, or sociology should first consider a course in that major, including ECON 2557 Economic Statistics, PSYC 2520 Data Analysis and SOC 2020 Quantitative Analysis in Sociology instead of MATH 1300 Introduction to Statistics and Computation or MATH 1756 Data Science.
- Students receiving a placement of either MATH 1700 Integral Calculus, MATH 1750 Intermediate Integral Calculus, MATH 1800 Multivariate Calculus, or MATH 2000 Linear Algebra and above who additionally have a year of high school or college biology are eligible to enroll in MATH 1758 Biomathematics. This course is appropriate for students interested in how differential calculus is used to address questions from biology.

### **Mathematics Placement Options**

- See Chair of the Mathematics Department: please contact P (https:// www.bowdoin.edu/profiles/faculty/jtaback/)rofessor Thomas Pietraho (https://www.bowdoin.edu/profiles/faculty/tpietrah/)
- See Director of Quantitative Reasoning: please contact Professor Eric Gaze (https://www.bowdoin.edu/profiles/faculty/egaze/)
- MATH 1050 Quantitative Reasoning is based on high school mathematical preparation and appropriate for students who may benefit from additional preparation before enrolling in further quantitative courses.
- MATH 1600 Differential Calculus is for students who have not yet seen calculus, or have seen up to one semester of calculus in high school.
- MATH 1700 Integral Calculus and MATH 1750 Intermediate Integral Calculus is for students who have had AB calculus or its equivalent in high school.
- MATH 1800 Multivariate Calculus is for students who have had BC calculus or its equivalent in high school. Student scores on the AP or IB exam do not affect this placement.
- MATH 2000 Linear Algebra/MATH 2020 Introduction to Mathematical Reasoning is recommended for students with advanced preparation.

These are courses for students who have already completed multivariate calculus. Students with this placement should attend the information session offered by the department outlining these courses.

#### **Statistics Placement Options**

- MATH 1300 Introduction to Statistics and Computation is an introduction to the statistical methods used in the life sciences. The course assumes minimal or no background in calculus or statistics.
  - Incoming students placed in MATH 1600 Differential Calculus or MATH 1700 Integral Calculus may enroll.
- MATH 1756 Data Science is an introduction to data science with an emphasis on the use of computational methods to explore, visualize, and contextualize data using a variety of statistical and probability models.
  - Incoming students placed in MATH 1750 Intermediate Integral Calculus or higher may enroll.
- MATH 1758 Biomathematics is a more comprehensive introduction to statistics as it is used across the natural and social sciences and assumes some background in calculus or statistics.
  - Incoming students placed in MATH 1700 Integral Calculus or higher may enroll.

# Courses

# MATH 1050 (a, MCSR) Quantitative Reasoning Eric Gaze.

Every Semester. Fall 2024. Enrollment limit: 30.

Explores the ways and means by which we communicate with numbers; the everyday math we encounter on a regular basis. The fundamental quantitative skill set is covered in depth providing a firm foundation for further coursework in mathematics and the sciences. Topics include ratios, rates, percentages, units, descriptive statistics, linear and exponential modeling, correlation, logic, and probability. A project-based course using Microsoft Excel, emphasizing conceptual understanding and application. Reading of current newspaper articles and exercises involving personal finance are incorporated to place the mathematics in real-world context.

Prerequisites: Placement in MATH 1050 or Placement in MATH 1050 (S/ M).

Previous terms offered: Spring 2024, Fall 2023, Spring 2023, Fall 2022, Spring 2022, Fall 2021, Fall 2020.

# MATH 1100 (a, MCSR) Introduction to Data Visualization Eric Gaze.

Every Spring. Spring 2025. Enrollment limit: 30.

The purpose of this course is to provide a comprehensive introduction to data analysis and data visualization. Students will use spreadsheet applications to analyze and interpret data before progressing to the more powerful tools of R and Tableau. Students will come to appreciate the ease and utility of spreadsheets, but also learn that for many projects, other software platforms provide more power, flexibility, and convenience, and produce better results. A main goal of the course is to transition students to a coding environment, overcoming the conceptual and practical hurdles involved in that transition. The course is organized around a set of case studies. For each data set, we set out to create the same graphics using each of three tools: Excel, Tableau, and R. Students will learn how to wrangle their data into a tidy form suitable for analysis and visualization. We will engage in all aspects of the data science process.

# MATH 1300 (a, MCSR) Introduction to Statistics and Computation Jack O'Brien.

Every Semester. Fall 2024; Spring 2025. Enrollment limit: 30.

An introduction to statistical methods used across the social and natural sciences with an emphasis on computational techniques. Covers conceptual understanding and includes topics from exploratory data analysis, the experimental design, probability, and statistical inference. Computational skills form a core element of the course, used throughout the semester both to explore data and to execute statistical tests. Not open to students who have credit for Economics 2557 or Psychology 2520 or who have credit for or are concurrently enrolled in Mathematics 1400.

Prerequisites: MATH 1050 or MATH 1051 or Placement in MATH 1300 or 1400 (S) or Placement in MATH 1300 (S) or Placement in MATH 1300 or 2206(S) or Placement in MATH 1600 (M) or Placement in MATH 1600 or Placement in MATH 1700 (M) or Placement in MATH 1700.

Previous terms offered: Spring 2024, Fall 2023, Spring 2023, Fall 2022, Fall 2021, Spring 2021, Fall 2020.

#### MATH 1600 (a, MCSR) Differential Calculus

Michael Ben-Zvi; Christopher Chong. Every Semester. Fall 2024; Spring 2025. Enrollment limit: 30.

Functions, including the trigonometric, exponential, and logarithmic functions; the derivative and the rules for differentiation; the anti-derivative; applications of the derivative and the anti-derivative. Four to five hours of class meetings and computer laboratory sessions per week, on average. Open to students who have taken at least three years of mathematics in secondary school.

Prerequisites: MATH 1050 or Placement in MATH 1600 (M) or Placement in MATH 1600 or PHYS 1093 (same as CHEM 1093).

Previous terms offered: Spring 2024, Fall 2023, Spring 2023, Fall 2022, Spring 2022, Fall 2021, Spring 2021, Fall 2020.

#### MATH 1610 (a) Emerging Scholars Differential Calculus Workshop Every Semester. Enrollment limit: 08. .5 Credit Credit/D/F Only.

Supplemental problem-solving workshop for differential calculus students. Concurrent enrollment in Math 1600 required. Enrollment by permission of instructor. One-half credit. Grading is Credit/D/Fail.

Prerequisites: Placement in MATH 1600 (M) or Placement in MATH 1600 or MATH 1600.

Previous terms offered: Spring 2024, Fall 2023, Spring 2023, Fall 2022.

# MATH 1700 (a, MCSR) Integral Calculus

Rhiannon Griffiths; Michael Ben-Zvi. Every Semester. Fall 2024; Spring 2025. Enrollment limit: 30.

The definite integral; the Fundamental theorems; improper integrals; applications of the definite integral; differential equations; and approximations including Taylor polynomials and Fourier series. An average of four to five hours of class meetings and computer laboratory sessions per week.

Prerequisites: MATH 1600 or Placement in MATH 1700 (M) or Placement in MATH 1700.

Previous terms offered: Spring 2024, Fall 2023, Spring 2023, Fall 2022, Spring 2022, Fall 2021, Spring 2021, Fall 2020.

#### MATH 1710 (a) Emerging Scholars Integral Calculus Workshop Every Semester. Enrollment limit: 08. .5 Credit Credit/D/F Only.

Supplemental problem-solving workshop for integral calculus students. Concurrent enrollment in Math 1700 required. Enrollment by permission of instructor. One-half credit. Grading is Credit/D/Fail.

Prerequisites: MATH 1600 or Placement in MATH 1700 (M) or Placement in MATH 1700.

Previous terms offered: Spring 2024, Fall 2023, Spring 2023, Fall 2022.

### MATH 1750 (a, MCSR) Intermediate Integral Calculus

Michael Ben-Zvi; Rhiannon Griffiths. Every Semester. Fall 2024; Spring 2025. Enrollment limit: 30.

A review of the exponential and logarithmic functions, techniques of integration, and numerical integration. Improper integrals. Approximations using Taylor polynomials and infinite series. Emphasis on differential equation models and their solutions. An average of four to five hours of class meetings and computer laboratory sessions per week. Open to students whose backgrounds include the equivalent of Mathematics 1600 and the first half of Mathematics 1700. Designed for first-year students who have completed an AB Advanced Placement calculus course in their secondary schools.

Prerequisites: Placement in MATH 1750 (M) or Placement in MATH 1750.

Previous terms offered: Spring 2024, Fall 2023, Spring 2023, Fall 2022, Spring 2022, Fall 2021.

# MATH 1756 (a, MCSR) Data Science

Jack O'Brien.

Every Spring. Fall 2024. Enrollment limit: 30.

An introduction to data science through computer programming. Emphasis on the use of computational methods to explore, visualize, and contextualize data using a variety of statistical and probability models. Readings from scientific literature are paired with techniques to interpret data in a variety of contexts. Topics include computer programming, data organization, exploratory data analysis, probability, random variables, statistical tests, regression, the use (and misuse) of p-values, and scientific argumentation. No previous programming experience is assumed. Not open to students who have credit for Economics 2557.

Prerequisites: Placement in MATH 1750 (M) or Placement in MATH 1800 (M) or Placement in MATH 2020 or 2206 (M) or Placement in 2000, 2020, 2206 (M) or Placement in MATH 1750 or Placement in MATH 1800 or Placement in MATH 2000 or 2020 or Placement in MATH 2020 or MATH 1700 - 2969.

#### MATH 1758 (a, MCSR) Biomathematics

Mary Lou Zeeman. Every Fall. Spring 2025. Enrollment limit: 30.

A study of mathematical modeling in biology, with a focus on translating back and forth between biological questions and their mathematical representation. Biological questions are drawn from a broad range of topics, including disease, ecology, genetics, population dynamics, and neurobiology. Mathematical methods include discrete and continuous (ODE) models and simulation, box models, linearization, stability analysis, attractors, oscillations, limiting behavior, feedback, and multiple time-scales. Within the biology major, this course may count as the mathematics credit or as biology credit, but not both. Students are expected to have taken a year of high school or college biology prior to this course. (Same as: BIOL 1175)

Prerequisites: MATH 1600 - 2969 or Placement in MATH 1700 (M) or Placement in MATH 1750 (M) or Placement in MATH 1800 (M) or Placement in MATH 2020 or 2206 (M) or Placement in 2000, 2020, 2206 (M) or Placement in MATH 1700 or Placement in MATH 1750 or Placement in MATH 1800 or Placement in MATH 2020 or 2020 or Placement in MATH 2020.

#### MATH 1800 (a, MCSR) Multivariate Calculus

Adam Levy; Mary Lou Zeeman; Rhiannon Griffiths. Every Semester. Fall 2024; Spring 2025. Enrollment limit: 30.

Multivariate calculus in two and three dimensions. Vectors and curves in two and three dimensions; partial and directional derivatives; the gradient; the chain rule in higher dimensions; double and triple integration; polar, cylindrical, and spherical coordinates; line integration; conservative vector fields; and Green's theorem. An average of four to five hours of class meetings and computer laboratory sessions per week.

Prerequisites: MATH 1700 or MATH 1750 or Placement in MATH 1800 (M) or Placement in MATH 1800.

Previous terms offered: Spring 2024, Fall 2023, Spring 2023, Fall 2022, Spring 2022, Fall 2021, Spring 2021, Fall 2020.

#### MATH 2000 (a, MCSR) Linear Algebra

Naomi Tanabe; Rhiannon Griffiths. Every Semester. Fall 2024; Spring 2025. Enrollment limit: 30.

A study of linear algebra in the context of Euclidean spaces and their subspaces, with selected examples drawn from more general vector spaces. Topics will include: vectors, linear independence and span, linear transformations, matrices and their inverses, bases, dimension and rank, determinants, eigenvalues and eigenvectors, diagonalization and change of basis, and orthogonality. Applications drawn from linear systems of equations, discrete dynamical systems, Markov chains, computer graphics, and least-squares approximation.

Prerequisites: MATH 1800 or Placement in 2000, 2020, 2206 (M) or Placement in MATH 2000 or 2020 or Placement in MATH 2020.

Previous terms offered: Spring 2024, Fall 2023, Spring 2023, Fall 2022, Spring 2022, Fall 2021, Spring 2021, Fall 2020.

MATH 2020 (a, MCSR) Introduction to Mathematical Reasoning Naomi Tanabe; Chandrika Sadanand; Michael Ben-Zvi. Every Semester. Fall 2024; Spring 2025. Enrollment limit: 20.

An introduction to logical deductive reasoning and mathematical proof through diverse topics in higher mathematics. Specific topics include set and function theory, modular arithmetic, proof by induction, and the cardinality of infinite sets. May also consider additional topics such as graph theory, number theory, and finite state automata.

Prerequisites: MATH 1800 or Placement in 2000, 2020, 2206 (M) or Placement in MATH 2020 or 2206 (M) or Placement in MATH 2000 or 2020 or Placement in MATH 2020.

Previous terms offered: Spring 2024, Fall 2023, Spring 2023, Fall 2022, Spring 2022, Fall 2021, Spring 2021, Fall 2020.

#### MATH 2109 (a, MCSR) Optimization

Adam Levy.

Every Other Spring. Fall 2024; Spring 2025. Enrollment limit: 30.

A study of optimization problems arising in a variety of situations in the social and natural sciences. Analytic and numerical methods are used to study problems in mathematical programming, including linear models, but with an emphasis on modern nonlinear models. Issues of duality and sensitivity to data perturbations are covered, and there are extensive applications to real-world problems.

Prerequisites: MATH 2000 or Placement in MATH 2020 or 2206 (M) or Placement in MATH 2020.

Previous terms offered: Spring 2021, Fall 2020.

#### MATH 2206 (a, MCSR) Probability

# Ray Maresca.

Every Semester. Fall 2024; Spring 2025. Enrollment limit: 35.

A study of the mathematical models used to formalize nondeterministic or "chance" phenomena. General topics include combinatorial models, probability spaces, conditional probability, discrete and continuous random variables, independence and expected values. Specific probability densities, such as the binomial, Poisson, exponential, and normal, are discussed in depth.

Prerequisites: MATH 2000 or Placement in MATH 2020 or 2206 (M) or Placement in 2000, 2020, 2206 (M) or Placement in MATH 2000 or 2020 or Placement in MATH 2020.

Previous terms offered: Fall 2023, Spring 2023, Fall 2022, Spring 2022, Fall 2021, Spring 2021, Fall 2020.

MATH 2208 (a, MCSR) Ordinary Differential Equations Christopher Chong; Mary Lou Zeeman. Every Semester. Fall 2024; Spring 2025. Enrollment limit: 30.

A study of some of the ordinary differential equations that model a variety of systems in the physical, natural and social sciences. Classical methods for solving differential equations with an emphasis on modern, qualitative techniques for studying the behavior of solutions to differential equations. Applications to the analysis of a broad set of topics, including population dynamics, oscillators and economic markets. Computer software is used as an important tool, but no prior programming background is assumed.

Prerequisites: MATH 2000 or Placement in MATH 2020 or 2206 (M) or Placement in 2000, 2020, 2206 (M) or Placement in MATH 2020.

Previous terms offered: Spring 2024, Fall 2023, Spring 2023, Fall 2022, Spring 2022, Fall 2021, Spring 2021, Fall 2020.

MATH 2209 (a, MCSR) Numerical Methods Every Other Spring. Enrollment limit: 30.

An introduction to the theory and application of numerical analysis. Topics include approximation theory, numerical integration and differentiation, iterative methods for solving equations, and numerical analysis of differential equations.

Prerequisites: MATH 2000 or Placement in MATH 2020 or 2206 (M) or Placement in 2000, 2020, 2206 (M) or Placement in MATH 2020.

Previous terms offered: Spring 2024, Spring 2022.

MATH 2301 (a, MCSR) Intermediate Linear Algebra Every Other Spring. Enrollment limit: 35.

A continuation of Linear Algebra focused on the interplay of algebra and geometry as well as mathematical theory and its applications. Topics include matrix decompositions, eigenvalues and spectral theory, vector and Hilbert spaces, norms and low-rank approximations. Applications to biology, computer science, economics, and statistics, including artificial learning and pattern recognition, principal component analysis, and stochastic systems. Course and laboratory work balanced between theory and application.

Prerequisites: Two of: MATH 2000 or Placement in MATH 2020 or 2206 (M) or Placement in MATH 2020 and MATH 2020.

Previous terms offered: Fall 2022.

### MATH 2303 (a, MCSR) Functions of a Complex Variable

Every Other Fall. Enrollment limit: 35.

The differential and integral calculus of functions of a complex variable. Cauchy's theorem and Cauchy's integral formula, power series, singularities, Taylor's theorem, Laurent's theorem, the residue calculus, harmonic functions, and conformal mapping.

Prerequisites: MATH 1800 or Placement in MATH 2020 or 2206 (M) or Placement in 2000, 2020, 2206 (M) or Placement in MATH 2000 or 2020 or Placement in MATH 2020.

Previous terms offered: Spring 2024, Spring 2022.

MATH 2404 (a, MCSR) Geometry Chandrika Sadanand. Every Other Spring. Spring 2025. Enrollment limit: 30.

A survey of modern approaches to Euclidean geometry in two dimensions. Axiomatic foundations of metric geometry. Transformational geometry: isometries and similarities. Klein's Erlanger Programm. Symmetric figures. Other topics may be chosen from three-dimensional geometry, ornamental groups, area, volume, fractional dimension, and fractals.

Prerequisites: MATH 2020.

Previous terms offered: Spring 2024, Spring 2023, Spring 2022, Fall 2020.

MATH 2502 (a, MCSR) Number Theory and Cryptography Every Other Spring. Enrollment limit: 30.

A survey of number theory from Euclid's proof that there are infinitely many primes through Wiles's proof of Fermat's Last Theorem in 1994. Prime numbers, unique prime factorization, and results on counting primes. The structure of modular number systems. Continued fractions and "best" approximations to irrational numbers. Investigation of the Gaussian integers and other generalizations. Squares, sums of squares, and the law of quadratic reciprocity. Applications to modern methods of cryptography, including public key cryptography and RSA encryption.

Prerequisites: MATH 2020.

Previous terms offered: Fall 2023, Spring 2021.

MATH 2601 (a, MCSR) Combinatorics and Graph Theory Every Other Year. Enrollment limit: 35.

An introduction to combinatorics and graph theory. Topics to be covered may include enumeration, matching theory, generating functions, partially ordered sets, Latin squares, designs, and graph algorithms.

Prerequisites: Two of: MATH 2020 and MATH 2000 or Placement in MATH 2020 or Placement in MATH 2020 or 2206 (M).

Previous terms offered: Spring 2023, Fall 2021.

## MATH 2602 (a, MCSR) Group Theory

Every Other Fall. Enrollment limit: 35.

An introduction to the theory of finite and infinite groups, with examples ranging from symmetry groups to groups of polynomials and matrices. Properties of mappings that preserve algebraic structures are studied. Topics include cyclic groups, homomorphisms and isomorphisms, normal subgroups, factor groups, the structure of finite abelian groups, and Sylow theorems.

Prerequisites: Two of: MATH 2000 or Placement in MATH 2020 or 2206 (M) or Placement in MATH 2020 and MATH 2020.

Previous terms offered: Fall 2022, Fall 2021.

MATH 2603 (a, MCSR) Introduction to Analysis Thomas Pietraho. Every Fall. Fall 2024. Enrollment limit: 35.

Building on the theoretical underpinnings of calculus, develops the rudiments of mathematical analysis. Concepts such as limits and convergence from calculus are made rigorous and extended to other contexts, such as spaces of functions. Specific topics include metric spaces, point-set topology, sequences and series, continuity, differentiability, the theory of Riemann integration, and functional approximation and convergence.

Prerequisites: Two of: MATH 2020 and MATH 2000 or Placement in MATH 2020 or 2206 (M) or Placement in MATH 2020.

Previous terms offered: Spring 2024, Fall 2022, Fall 2021, Spring 2021.

MATH 2606 (a, MCSR) Statistics Jack O'Brien.

Every Spring. Spring 2025. Enrollment limit: 35.

An introduction to the fundamentals of mathematical statistics. General topics include likelihood methods, point and interval estimation, and tests of significance. Applications include inference about binomial, Poisson, and exponential models, frequency data, and analysis of normal measurements.

Prerequisites: MATH 2206.

Previous terms offered: Spring 2024, Spring 2023, Spring 2022, Spring 2021.

MATH 2702 (a, MCSR) Rings and Fields

Ray Maresca.

Every Other Fall. Fall 2024. Enrollment limit: 35.

An introduction to algebraic structures based on the study of rings and fields. Structure of groups, rings, and fields, with an emphasis on examples. Fundamental topics include: homomorphisms, ideals, quotient rings, integral domains, polynomial rings, field extensions. Further topics may include unique factorization domains, rings of fractions, finite fields, vector spaces over arbitrary fields, and modules. Mathematics 2502 is helpful but not required.

Prerequisites: Two of: MATH 2000 or Placement in MATH 2020 or 2206 (M) or Placement in 2000, 2020, 2206 (M) or Placement in MATH 2020 and MATH 2020.

Previous terms offered: Fall 2023, Fall 2020.

# MATH 2805 (a, MCSR) Mathematical principles of machine learning Thomas Pietraho.

Every Other Spring. Fall 2024. Enrollment limit: 30.

An introduction to the mathematical theory and practice of machine learning. Supervised and unsupervised learning problems, including regression, classification, clustering, and component analysis, focusing on techniques most relevant to the study and applications of neural networks. Additional topics may include dimension reduction, data visualization, denoising, norms and loss functions, optimization, universal approximation theorems, and algorithmic fairness. Class will include computer lab and projects, but no formal programming experience is necessary.

Prerequisites: MATH 2603.

Previous terms offered: Spring 2023, Spring 2022.

#### MATH 3108 (a) Advanced Topics in Modeling Every Other Spring. Enrollment limit: 16.

A study of mathematical modeling, with emphasis on how to identify scientific questions appropriate for modeling, how to develop a model appropriate for a given scientific question, and how to interpret model predictions. Applications drawn from the natural, physical, environmental, and sustainability sciences. Model analysis uses a combination of computer simulation and theoretical methods and focuses on predictive capacity of a model.

Prerequisites: Three of: MATH 2000 or Placement in MATH 2020 or 2206 (M) or Placement in 2000, 2020, 2206 (M) or Placement in MATH 2020 and MATH 2020 and MATH 2208.

Previous terms offered: Spring 2024, Spring 2022.

#### MATH 3109 (a) Optimal Control

Every Other Fall. Enrollment limit: 16.

A study of infinite-dimensional optimization, including calculus of variations and optimal control. Classical, analytic techniques are covered, as well as numerical methods for solving optimal control problems. Applications in many topic areas, including economics, biology, and robotics.

Prerequisites: Three of: MATH 2000 and MATH 2020 and MATH 2208.

Previous terms offered: Fall 2023, Fall 2021.

#### MATH 3202 (a) Advanced Topics in Algebra Ray Maresca.

Every Other Year. Spring 2025. Enrollment limit: 16.

One or more specialized topics in abstract or linear algebra. Possible topics include: representations and modules, quivers, categories, homological algebra, spectral and operator theory, and algebraic combinatorics, as well as others that explore the connection of algebra with other areas of modern mathematics.

Prerequisites: Three of: MATH 2000 or Placement in MATH 2020 or 2206 (M) or Placement in MATH 2020 and MATH 2020 and either MATH 2602 or MATH 2603 or MATH 2702.

# MATH 3204 (a) Topology

Chandrika Sadanand. Every Other Fall. Fall 2024. Enrollment limit: 16.

A mathematical study of shape. Examination of surfaces, knots, and manifolds with or without boundary. Topics drawn from point-set topology, algebraic topology, knot theory, and computational topology, with possible applications to differential equations, graph theory, topological data analysis, and the sciences.

Prerequisites: Three of: MATH 2000 or Placement in MATH 2020 or 2206 (M) or Placement in MATH 2020 and MATH 2020 and either MATH 2602 or MATH 2603 or MATH 2702.

Previous terms offered: Fall 2022, Spring 2021.

#### MATH 3208 (a) Advanced Topics in Dynamical Systems Every Other Spring. Enrollment limit: 16.

A study of nonlinear dynamical systems arising in applications, with emphasis on modern geometric, topological, and analytical techniques to determine global system behavior, from which predictions can be made. Topics chosen from local stability theory and invariant manifolds, limit cycles and oscillation, global phase portraits, bifurcation and resilience, multiple time scales, and chaos. Theoretical methods supported by simulations. Applications drawn from across the sciences.

Prerequisites: Four of: MATH 2000 or Placement in MATH 2020 or 2206 (M) or Placement in 2000, 2020, 2206 (M) or Placement in MATH 2020 and MATH 2020 and MATH 2020 and MATH 2603.

Previous terms offered: Fall 2022.

#### MATH 3209 (a) Partial Differential Equations Christopher Chong.

Every Other Fall. Fall 2024; Spring 2025. Enrollment limit: 16.

A study of some of the partial differential equations that model a variety of systems in the natural and social sciences. Classical methods for solving partial differential equations are covered, as well as modern, numerical techniques for approximating solutions. Applications to the analysis of a broad set of topics, including air quality, traffic flow, and imaging. Computer software is used as an important tool.

Prerequisites: Three of: MATH 2000 or Placement in MATH 2020 or 2206 (M) or Placement in 2000, 2020, 2206 (M) or Placement in MATH 2020 and MATH 2020 and MATH 2208.

Previous terms offered: Spring 2023, Spring 2021, Fall 2020.

MATH 3303 (a, MCSR) Advanced Complex Analysis Every Other Spring. Enrollment limit: 16.

A second course in complex analysis. Topics may include conformal mappings, harmonic functions, and analytic functions. Applications drawn from boundary value problems, elliptic functions, two-dimensional potential theory, Fourier analysis, and topics in analytic number theory.

Prerequisites: Three of: MATH 2020 and MATH 2303 and MATH 2000 or Placement in MATH 2020 or 2206 (M) or Placement in MATH 2020.

Previous terms offered: Fall 2022.

#### MATH 3404 (a) Advanced Topics in Geometry

Every Other Fall. Enrollment limit: 16.

An introduction to advanced topics in geometry, including Euclidean and non Euclidean geometries in two dimensions, unified by the transformational viewpoint of Klein's Erlangen Program. Additional topics will vary by instructor and may include isometry groups of Euclidean and hyperbolic spaces, alternate models of hyperbolic geometry, differential geometry and projective geometry. Math 2404, or any higher numbered course, is helpful but not required.

Prerequisites: Three of: MATH 2000 or Placement in MATH 2020 or 2206 (M) or Placement in 2000, 2020, 2206 (M) or Placement in MATH 2020 and MATH 2020 and either MATH 2301 or MATH 2404 or MATH 2502 or MATH 2601 or MATH 2602 or MATH 2603 or MATH 2702.

Previous terms offered: Fall 2023, Fall 2021, Fall 2020.

MATH 3602 (a) Advanced Topics in Group Theory Every Other Spring. Enrollment limit: 16.

The study of group actions on geometric objects; understanding finite and discrete groups via generators and presentations. Applications to geometry, topology, and linear algebra, focusing on certain families of groups. Topics may include Cayley graphs, the word problem, growth of groups, and group representations.

Prerequisites: Three of: MATH 2602 and MATH 2020 and MATH 2000 or Placement in MATH 2020 or 2206 (M) or Placement in MATH 2020.

Previous terms offered: Spring 2023, Spring 2022.

#### MATH 3603 (a) Advanced Analysis Thomas Pietraho.

Every Other Spring. Spring 2025. Enrollment limit: 16.

Measure theory and integration with applications to probability and mathematical finance. Topics include Lebesgue measure and integral, measurable functions and random variables, convergence theorems, analysis of random processes including random walks and Brownian motion, and the Ito integral.

Prerequisites: Three of: MATH 2020 and MATH 2603 and MATH 2000 or Placement in MATH 2020 or 2206 (M) or Placement in MATH 2020.

Previous terms offered: Spring 2022.

MATH 3606 (a) Advanced Topics in Probability and Statistics Every Other Fall. Enrollment limit: 16.

One or more specialized topics in probability and statistics. Possible topics include regression analysis, nonparametric statistics, logistic regression, and other linear and nonlinear approaches to modeling data. Emphasis is on the mathematical derivation of the statistical procedures and on the application of the statistical theory to real-life problems.

Prerequisites: Three of: MATH 2020 and MATH 2606 and MATH 2000 or Placement in MATH 2020 or 2206 (M) or Placement in MATH 2020.

Previous terms offered: Spring 2024, Fall 2023, Fall 2021.

#### MATH 3702 (a) Advanced Topics in Rings and Number Theory Every Other Spring. Enrollment limit: 16.

Advanced topics in modern algebra based on rings and fields. Possible topics include: Galois theory with applications to geometric constructions and (in)solvability of polynomial equations; algebraic number theory and number fields such as the p-adic number system; commutative algebra; algebraic geometry and solutions to systems of polynomial equations.

Prerequisites: Three of: MATH 2020 and MATH 2702 and MATH 2000 or Placement in MATH 2020 or 2206 (M) or Placement in MATH 2020.

Previous terms offered: Spring 2024, Spring 2021.